Using a Soft Deadline to Counter Monopoly *

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Abstract

A monopolist often exploits a hard deadline to raise their commitment power. I explore whether a group of buyers can employ a soft deadline to counter the monopoly. Using a simple durable goods monopolist model under a deadline, I show that the buyers' imperfect commitment to an earlier exit may elicit a compromise from the monopolist and generate the buyers' premium. The soft deadline partially restores the self-competition dynamics of Coase conjecture, which is previously constrained by the hard deadline. Only one soft deadline breaks the conventional link between the time horizon (or durability of goods) and monopoly power.

JEL Classification: C78, C91

Keywords: bargaining, durable goods monopoly, commitment, Coase conjecture, deadline effect

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1 Introduction

Across the developed economies, the market power of leading firms is rising. (See e.g. De Loecker and Eeckhout (2018) for the global evolution of market power and Autor et al. (2020) for the rise of superstar firms in the U.S.) Bilateral bargaining typically forms their mode of trade from procurement contracts, business partnerships, corporate acquisitions and labor group disputes. For negotiators in the industrial economy, reinforcing their bargaining power against monopolists is of central importance. Observing that most influential modern monopolists sell durable goods (e.g. software, intellectual property, semiconductors, natural resources), a workhorse framework well-suited for the world is a durable goods monopolist model (Stokey (1981); Bulow (1982)). In this scenario, the monopolist sells his goods to a demand pool of buyers with their private value. The model is widely applied to outside the seller-buyer trades; labor group disputes (Hart (1989)), medical malpractice disputes (Sieg (2000)), sovereign debt renegotiation (Bai and Zhang (2012)), and hostage-taking negotiation with pirates (Ambrus, Chaney and Salitskiy (2018)). Behind the framework, the long-known Coase conjecture (proposed by Coase (1972), later formalized by Gul, Sonnenschein and Wilson (1986)) lies as the theoretical cornerstone. The theory essentially states that the monopolist loses bargaining power via self-competition when the buyer rationally expects his future concession from the irresistible temptation for price discrimination. Substantial theoretical attempts have been made to revive the monopolist's commitment, by depreciating goods (Bond and Samuelson (1984)), discrete demands (Bagnoli, Salant and Swierzbinski (1989)), the arrival of new buyers (Fuchs and Skrzypacz (2010)), and the buyer's outside options (Board and Pycia (2014)).

One natural tactic to circumvent the conjecture is commitment to a deadline (Sobel and Takahashi (1983); Fudenberg, Levine and Tirole (1985)). Casual observation suggests that monopolists maintain a stronger reputational concern by imposing a perfectly committed hard deadline. Armed with the deadline, even as the proposer's commitment to the offer disappears (i.e., a length of each bargaining round), the proposer regains his bargaining power by framing the offer as an "ultimatum". (See, e.g., Fershtman and Seidmann (1993); Güth and Ritzberger (1998))[1] In this paper, I question whether a deadline is an exclusive commitment device for the monopolist. Each buyer's reputational concern is plausibly weaker than the monopolist.

[1] Gneezy, Haruvy and Roth (2003) consider an infinite-horizon version of ultimatum game where the proposer completely loses the bargaining power, and call it a "reverse" ultimatum game. However, the simple imposition of the deadline completely recovers the power, making the game resemble a canonical ultimatum game.

I therefore explore whether a group of buyers could partially commit to an intermediate *soft* deadline to counter the monopoly power recovered by the hard deadline under the classical context of a durable goods monopoly.

Soft deadlines have been implemented or may well be implemented among a variety of collective bargaining disputes in the real world. For instance, in a procurement for scarce resources (e.g. gasoline, electricity, natural resources, semiconductors), a group of small-sized firms or municipalities very often form a purchase consortium to negotiate with a monopolistic—occasionally foreign—supplier.[2] (Regarding purchasing consortia, see, e.g. Glock and Hochrein (2011); Tella and Virolainen (2005). For collaborative procurement in the public sector, see Walker et al. (2013).) This consortium of small firms might reinforce their bargaining power by partially committing to earlier switching to another supplier. In a labor union negotiation, a union bargains over a wage under a publicly announced deadline regarding when a strike starts (see, e.g. Card (1990); Cramton and Tracy (1992)). The union may dangle the possibility of an earlier strike, conditional on a sluggish negotiation progress.[3] In another example, in negotiations to release the captured hostages, terror groups or pirates often demand the payment of ransom within a deadline.[4] The government or the police serving in a diplomatic role often issue a warning of earlier suppression by force before the deadline. I formally illustrate these scenarios by naturally extending a simple Coasian model under a deadline and show that the buyers' imperfect commitment to an earlier exit would create a novel motive for price discrimination, and thus eliciting a concession from the monopolist.

I start with the canonical bargaining model of a durable goods monopolist. Consider that a monopolist (he) sells his goods of zero marginal costs to a continuum of buyers (she) and imposes a perfectly committed hard deadline at time T. A single buyer's valuation is private information, but the value distribution is shared as common knowledge. Each buyer rejects

[2]A large conglomerate firm sometimes runs a collective procurement instead of an independent purchase by its subsidiary. Under the recent global shortage of semiconductors, for example, Toyota (a large auto company) launched a group purchase (Daily Industrial Newspaper, April 21, 2023).

[3]Aside from the industrial world, a soft deadline commitment has been applied in the international sovereign debt negotiations. Greece held a national referendum before the default deadline to accept the bailout proposals by the EC, ECB and IMF in 2015. Although it entailed no legal binding power, the analysts predicted that Greece's rejection might trigger a chain reaction of financial terror. (*The Teregraph*, July 6, 2015)

[4]Ransom remains to be a vital source of funding for terrorist groups and pirates. For example, the terror group al-Qaeda obtained 125 million dollars in ransom from kidnapping during 2008–2014 (New York Times, July 30, 2014). In addition, pirates in Somalia received 360 million dollars during 2005–2012 (Yikona (2013)). A dataset by Mickolus et al. (1976) records the requested ransom and imposed deadlines in the history of terrorist incidents.

every offer until the offer is accepted. When the deadline passes, both get 0. In the unique equilibrium, the asymmetric information generates a delay until agreement to screen the buyer; the price schedule is declining overtime, and a lower-type buyer wait longer for a discounted price.

In this environment, monopolistic power is hindered by two forces. First, the monopolist performs an intertemporal price discrimination endowed with multiple rounds of price offers. Second, the monopolist expects the buyer's incentive to wait and discounts the price under the strategic interaction. Consequently without a deadline, self-competition occurs, as conjectured by Coase (1972).[5] A hard deadline constraints the both forces: When the deadline approaches, a buyer is less likely to wait. Thus, the monopolist's offer resembles an ultimatum.

Suppose that before the bargaining starts, a group of buyers announces a soft deadline at a specific earlier time before the hard deadline, which I call a *threat point* just after the time t^* (see Figure 1). To maximize the expected surplus aggregated from the demand pool, the

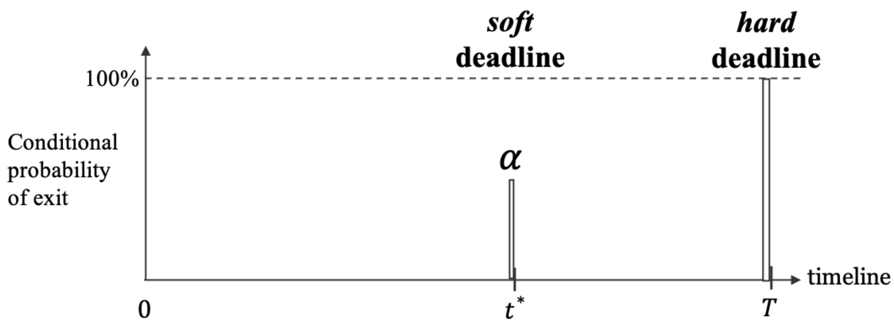

Figure 1: *soft* vs. *hard* deadline

Note: A *soft* deadline with a conditional probability of breakdown $\alpha \in (0,1)$ is imposed at time t^* in contrast to the *hard* deadline at time T. The buyer's group commits that the bargaining might end at t^* with probability α but continue with probability $1-\alpha$. In the main text below, I allow for multiple *soft* deadlines for generality.

group publicly announces its level of commitment to the deadline, characterized by a conditional probability of exit α, if the bargaining exceeds that point. My soft deadline framework nests a hard deadline setting as $\alpha = 0$ or 1. Intuitively, this soft deadline serves as an uncertain "time bomb" where the lower-type buyers who have not yet agreed to an offer would stochastically leave the bargaining table. (i.e., fall back to their outside options 0). One may view this commitment technology as an application of uncertain commitment (initially proposed by

[5] Güth (1994) called the dynamics as the intrapersonal price competition of the monopolist. In the language of screening-type bargaining, if an uninformed proposer has more frequent opportunities to revise the offers, and the informed responder rationally expects a future concession, the proposer loses the commitment power of offers.

Crawford (1982), and later, investigated by Ellingsen and Miettinen (2008)) to the deadline.

Consistent with the literature with a hard deadline with *deadline effect* (as tested in Roth, Murnighan and Schoumaker (1988) and formalized in Fershtman and Seidmann (1993)), I first show that more agreements are likely to occur just before the soft deadline.[6] This compromise on the buyers' side is framed as a stochastic analog of deadline effect. Within the strategic interaction, the monopolist exploits the buyers' compromise in an analogous way as an ultimatum game. More intriguingly, however, I show that the surprisingly subtle imposition of the soft deadline elicits the monopolist's price discrimination in a non-obvious way, but with intuitive appeal.

When the soft deadline safely passes ($t > t^*$), I show that the monopolist sharply performs a compromise, characterized as an atom of price cut. (See Figure 2.) This major sale is a new result of my model; occurring because if the buyer keeps rejecting at the threat point, she credibly signals that her value is substantially low. This is a direct consequence from the buyers' compromise at the threat point. Rejection at the soft deadline signals that agreement before the "time bomb" is still not profitable enough for the remaining buyer, and the monopolist is tempted to price discriminate. After the big sale, a remaining buyer benefits from the monopolist's discount. Expecting the less lucrative market after the soft deadline, the forward-looking monopolist may respond by driving down the price schedule ($t \leq t^*$) with a cheaper opening price at $t = 0$ to raise the probability of agreement before the soft deadline. (See Figure 2.) One may view a soft deadline as partially restoring the self-competition dynamics of Coase conjecture previously constrained by the hard deadline.[7]

At the soft deadline, both the compromise from price discrimination and exploitation from strategic interaction are at work. As both effects are counteracted, I formally demonstrate that the buyers' reasonably imperfect commitment potentially yields an aggregate premium. If the commitment is too hard, however, the soft deadline resembles a hard deadline, and the normal strategic interaction dominates and just backfires to augment the monopolist's power. I show that there exists an optimal interior commitment $\widehat{\alpha} \in (0,1)$ to the deadline, maximizing the

[6]In a market with a continuum value of buyers, this phenomenon is typically interpreted as a disproportionate agglomeration of purchases.

[7]In contrast to the Coase conjecture, the Packman conjecture (see, e.g. Bagnoli, Salant and Swierzbinski (1989)) claims that price discrimination is a source of monopoly power. Although this theory holds true for other bargaining protocols (e.g. complete information. See e.g. Von der Fehr and Kühn (1995)), this paper adopts Coase's view that myopic price discrimination undermines ex-ante monopoly power by hurting the commitment to pricing.

expected surplus of the buyers. Interestingly, under some parameter values, buyers of *all* types are shown to be better-off because of, or at least indifferent to, the imperfect commitment, assuring a participation constraint for a group to impose this commitment. (See Figure 4)

The finding casts a new light on the conventional wisdom regarding the bargaining horizon (or the typical interpretation as the durability of goods) and monopoly power under the durable goods monopoly. The Coase conjecture claims that durability reduces monopoly power, indicating that as the bargaining horizon expands, the surplus division is in favor of the buyers and the market becomes more efficient.[8] To see this claim intuitively, consider two polar cases. Under a one-shot time, the game is reduced to an ultimatum game with one-sided incomplete information.[9] The buyers suffer most under the strongest monopolist's commitment. In contrast, in an infinite horizon scenario, the monopolist is endowed with limitless opportunities of offers. Buyers enjoy competitive pricing and the bargaining is Pareto-efficient aligned with the Coase conjecture. In a more generic time structure allowing for soft deadlines, the finding suggests that bargaining appears *shorter*, but the buyers are better off in expectation.

Literature review A huge body of the theoretical bargaining literature has proposed the tactics to augment bargaining power, such as securing outside options (Compte and Jehiel (2002); Fuchs and Skrzypacz (2013); Hwang and Li (2017)), cheap talk (Crawford and Sobel (1982). See Farrell and Rabin (1996) for a survey.), and reputation from strategic irrationality (Abreu and Gul (2000); Kambe (1999)). Most relevant to my study is the literature on commitment initially ideated by Schelling (1960), and later formalized by Crawford (1982), Muthoo (1996), and Ellingsen and Miettinen (2008). Much of the literature stochastically frames credibility of commitment on the proposer's offer with some probability (α in my model), and very few examines the use of a deadline as a commitment technology.

The paper is also related to bargaining models with a strategic use of a given deadline. (Ma and Manove (1993); Fershtman and Seidmann (1993)). These studies chiefly explore the proposer's manipulation of delaying an offer with an intention to frame it as an ultimatum.[10]

[8] See, e.g., Güth and Ritzberger (1998).

[9] The ultimatum game is a two-stage game where a proposer and a responder bargain over a fixed money. In the first stage, the proposer offers his share, and in the second stage, the responder accepts or rejects it. If it is accepted, each receives money based on the offer; if rejected, each gets 0. A key difference from my model is that the game has complete information.

[10] The intuition echoes the revenue management literature (e.g., Horner and Samuelson (2011)), where the durable goods are perishable at the deadline. The key difference is that they limit the stocks of the goods, whereas a durable goods monopolist model is not subject to such limits.

Instead of strategic timing of the offer, my model adds the possibility of an exit at a specific event before the deadline. Using the reputation approach with the war of attrition protocol of Kambe (1999), Özyurt (2023) jointly analyzes the commitment to the offer of the proposer and the endogenous timing of the deadline, showing that the deadline setter is better off under the efficient unique equilibrium. This paper features the credibility of the deadline instead of its timing.

This study also builds on theoretical bargaining works on the role of hard deadlines on last-minute agreements (or the "deadline effect"). In the context of pre-trial negotiations, Spier (1992) uses a one-sided incomplete-information model similar to mine, deriving an agglomeration of trade at the deadline of a trial date. Ponsati (1995) and Damiano, Li and Suen (2012) derived a similar atom of trade at the hard deadline in concession games with two-sided incomplete information. My paper demonstrated that deadline effect occurs even if the deadline is soft. Fanning (2016) uses the reputation model of Abreu and Gul (2000) to provide a foundation of deadline effects from reputation across a wide range of protocols. In his model, however, one-sided incomplete information in the durable goods monopoly generates no delay.[11]

The main result of my paper (Proposition 4) contributes to the classic literature on the durable goods monopolist model (Stokey (1981), Bulow (1982), Gul, Sonnenschein and Wilson (1986)). Most of the literature adopts an infinite horizon framework. The avoidance of the zero-profit trap in the finite horizon framework is contained by Stokey (1979), Sobel and Takahashi (1983), and Fudenberg and Tirole (1983), although the role of a deadline is not explicitly mentioned. [12] Real-world bargainings often entails deadlines, and a series of applied works adopt a finite-horizon framework (e.g. labor union disputes for Tracy (1987), medical malpractice disputes for Sieg (2000), sovereign debt renegotiation for Bai and Zhang (2012)). All the durable goods monopoly literature in theoretical or applied works inherits the accepted wisdom launched in a seminar paper Coase (1972) titled "durability and monopoly", indicating that a shorter horizon (or less durability of goods) augments the monopoly power.[13] The key novelty is that adding only one soft deadline would break the conventional relationship between

[11] See the "Related Literature" section in Fanning (2016) for a comparison with one-sided, incomplete information models (e.g. Spier (1992); Fuchs and Skrzypacz (2013)).

[12] Ostatnickỳ (2004) emphasizes that a hard deadline fails the Coase conjecture, exploring the effect of discounting and decreasing private value to partially recover the Coase conjecture.

[13] Later studies formalize the relationship. (e.g.; Sobel and Takahashi (1983), Theorem 6, or Güth and Ritzberger (1998)). In the infinite horizon model, Stokey (1981) delivers a similar intuition; a longer time between offers (or less frequent offer revision) leads to a monopolist's gain.

the time horizon and monopoly power.

Though outside the durable goods monopoly context, the closest precedent to using a deadline in the continuous time is Fuchs and Skrzypacz (2013). In general, solving finite horizon models in the continuous limit is technically challenging, because the agent faces different time horizons at each decision under a non-stationary time structure. For the sake of tractability, I borrow their proof technique, rooted in Ausubel and Deneckere (1992). Conceptually, my paper diverges from Fuchs and Skrzypacz (2013) in three crucial dimensions. First, Fuchs and Skrzypacz explore the effect of relative outside options after a hard deadline on deadline effects, while my model highlights the role of a soft deadline on monopoly power, standardizing the outside options. Second, I show that a deadline effect also exists in a more general deadline structure. Third, my model delivers the new prediction of an atom price drop as a result of emerging price discrimination.

The characterization of soft deadlines is isomorphic to the idea of a random breakdown. Binmore, Rubinstein and Wolinsky (1986) introduced the risk of breakdown in the alternating-offers model a la Rubinstein (1982). Rubinstein and Wolinsky (1985) embed the risk in the model of decentralized market. The early models include no asymmetric information, and thus, generating no costly delays in equilibrium. However, later studies build models with incomplete information and random breakdown, where the *timing* of breakdown is uncertain under a continuous deadline distribution (Fuchs and Skrzypacz (2010); Fanning (2016); Simsek and Yildiz (2016)). By contrast, my model presumes that an occurrence of breakdown is uncertain for a specific event, contrived to capture a realistic feature of bargaining institutions. If breakdown comes with a continuous arrival rate, the magnitude of agglomeration of trades (deadline effects) or the price drop cannot be measured by atoms as in my paper (See Lemma 1 and Proposition 2).

On the technical front, a random breakdown is typically implemented as changing discounting factors. This formulation is also adopted by models with a stochastic arrival of deadlines in the continuous time, where a discount rate is adjusted by an arrival rate of a deadline. Mathematically, a discount factor is adjusted in my discrete period model to capture the credibility of the soft deadline. Meanwhile, in the limit case, a discount rate is not adjusted. Most of the bargaining literature deviating from the solo discount factors explores how the relative patience of both parties alters the surplus division (see, e.g., Binmore, Rubinstein and Wolinsky (1986);

Chapter 4.4 in Muthoo (1999)).[14] One could also view that my soft-deadline framework presumes a common patience for both parties, while allowing for more dynamically-varying patience (e.g. hyperbolic discounting).

Outline The paper is organized as follows: Section 2 presents a bargaining framework with an imperfectly committed deadline and characterizes the unique equilibrium. Section 3 shows that the monopolist performs a compromise before and after the threat point. Next, Section 4 demonstrates that the buyer's side enjoys a premium. The sensitivity of the overall efficiency is explored regarding the commitment intensity of the buyer's side. Section 5 concludes the paper. Most of the proofs of the results are provided in the Appendix.

2 Model

I start with a durable goods monopolist model, where an uninformed monopolist screens a pool of buyers under a hard deadline. I state the results in continuous time for simplicity although most of the results in the discrete period version remain unchanged.

2.1 Setup

A monopolist (he) negotiates with a buyer (she) to sell an indivisible good. Both are risk-neutral and forward-looking expected utility maximizers. The buyer has her private value $v \in [0,1]$ for the good and I assume that v is distributed according to the shared cumulative distribution function $F(v) = v^{\sigma}$ $(\sigma > 0)$.[15]

The monopolist's marginal cost is normalized to 0, and it is common knowledge.[16] Time is continuous with $t \in [0,T]$ and broken into a length $\Delta > 0$ for each bargaining round. Suppose that the monopolist credibly imposes a hard deadline at the time T. At the beginning of the time t, the monopolist proposes an offer p_t.[17] Then, the buyer immediately either accepts or

[14] For a durable goods context, Guth and Ritzberger (1992) explore that the relative patience of monopolists vs. consumers shapes the surplus division.

[15] This distributional assumption yields two benefits. First, the distribution shape is unchanged with Bayesian updating, which offers closed-form solutions by solving backwards. (Ausubel and Deneckere (1992), Fuchs and Skrzypacz (2013)) Second, it captures the first-order stochastic dominance. I conjecture that the qualitative predictions of the model are insensitive for any atomless and full-support distributions.

[16] This corresponds to the "no-gap" case, where a marginal cost is no lower than the lower bound of the buyer's private value.

[17] Ausubel and Deneckere (1992) provides a justification for the rule in which only the uninformed party is per-

rejects.[18] If she accepts the price at the time t, the game ends with this outcome: the monopolist gets $e^{-\gamma t} p_t$, and the buyer gets $e^{-\gamma t}(v - p_t)$, where γ is a common instantaneous discount rate. If she keep rejecting the price until $t = T$, both get 0 as an outside option.

Suppose that the buyers form a group to maximize their expected payoff, and the group imposes a series of soft deadlines at an earlier time $t_d \in (0,T)$ $(d \in \{1,\cdots,N\})$ with some imperfect commitment, captured by a conditional breakdown risk $\alpha_d \in (0,1)$ at the end of the time t_d.[19] When $N = 1$, I omit d for simplicity and write it as t^*. (e.g. Figure 1) This implies that if the proposal is rejected at time t_d, the bargaining ends with probability α_d and both get 0, but it proceeds to time $t_d + \Delta$ with probability $1 - \alpha_d$.[20] Observe that if $\alpha_d = 1$ for some d, the soft deadline is reduced to a hard deadline.

2.2 Equilibrium

What follows is a straightforward application of the canonical bargaining with incomplete information. (See Fudenberg, Levine and Tirole (1985) for the theoretical foundation.) Let $[0, k_t)$ be a posterior valuation at time t, and both players share $\{k_t\}$ as a belief system. Then, the buyer adopts this specific form of cutoff strategy: given k_t and p_t, she calculates the cutoff value c_t, satisfying the necessary condition for optimality:[21]

$$\underbrace{c_t - p_t}_{\text{payoff of agreement today}} = \underbrace{\eta_t e^{-\gamma\Delta}(c_t - p_{t+\Delta})}_{\text{payoff of agreement tomorrow}} .$$

Intuitively, the marginal buyer with a value $v = c_t$ is indifferent between buying today or tomorrow. Then, at time t, the buyer accepts p_t if $v \geq c_t$, and rejects if $v < c_t$. Immediately from this cutoff strategy, the belief system for both players is characterized by the cutoff schedule

$$c_t = k_{t+\Delta} \ \ (\forall t \in [0,\ T - \Delta]), \tag{1}$$

mitted to make offers. They showed that under alternating-offer games with one-sided incomplete information, the informed party endogenously never makes any serious offers when Δ is sufficiently short (the Silence Theorem).

[18] They are allowed to use mixed strategies, but this does not change the argument because the buyer's mixed strategy is rationalizable only when the private value is equal to the cutoff.

[19] Their participation constraint for the group is discussed below.

[20] This is different from cheap talk (see Farrell and Rabin (1996) for a survey) in that the group commits to the realization of breakdown. The setting is in line with bargaining with breakdown (e.g., Rubinstein and Wolinsky (1985); Binmore, Rubinstein and Wolinsky (1986)) as discussed in the literature review.

[21] Note that c_t is a function of p_t, not of v. Also, this necessary condition turns out to be sufficient due to the skimming property (i.e., the higher types agree earlier than the lower types). One can check that the difference between both hands is strictly increasing in c_t.

suggesting that the cutoff today is a supremum value of the buyer tomorrow. Then, the problem of the monopolist and the buyers' group is recursively defined as follows:

$$V_t = \max_{p_t} \underbrace{(\frac{F(k_t)-F(c_t)}{F(k_t)})}_{\text{probability of agreement}} p_t + \underbrace{\frac{F(c_t)}{F(k_t)}}_{\text{probability of rejection}} \eta_t e^{-\gamma\Delta} V_{t+\Delta} \tag{2}$$

$$W_t = \max_{c_t} \underbrace{(\frac{F(k_t)-F(c_t)}{F(k_t)})}_{\text{probability of agreement}} \{\mathbb{E}(v \mid c_t \leq v < k_t) - p_t\} + \underbrace{\frac{F(c_t)}{F(k_t)}}_{\text{probability of rejection}} \eta_t e^{-\gamma\Delta} W_{t+\Delta} \tag{3}$$

where V_t and W_t are value functions of each party, [22] and η_t is a risk adjustment factor attached to a discount factor $e^{-\gamma\Delta}$ such that

$$\eta_t = \begin{cases} 1-\alpha_d & (t = t_d) \\ 1 & (t \neq t_d) \end{cases}. \tag{4}$$

Mathematically, this is nothing but adjusting a discount factor in the particular time points t_d. I show below that this surprisingly simple formulation of commitment intensity regarding the deadline generates a perhaps unintended consequence for surplus division.[23] Lastly, an equilibrium is defined as a standard perfect Bayesian equilibrium.

Definition 1. *A pair of strategies $\{(p_t, c_t)\}$ and a belief system $\{k_t\}$ constitutes a perfect Bayesian equilibrium of the game if their actions maximize their expected payoffs at all information sets and a belief system is consistent with Bayes rule whenever possible.*

2.3 Dynamic Schedules

I solve the model backward. (See Sobel and Takahashi (1983) and the appendix in Fuchs and Skrzypacz (2013)) Given the state variable $\{k_t\}$ at time t, the equilibrium path of $\{(p_t, c_t)\}$ ($t \in [0, T]$) is sequentially characterized by $\{a_t\}$ and $\{b_t\}$ as follows:

$$p_t = a_t k_t \text{ and } c_t = b_t p_t \tag{5}$$

[22] W_t is interpreted as the collective buyer's continuation value at the time t.

[23] One can see this protocol adopts a general time discounting, including hyperbolic discounting and non-geometric discounting.

where $\{a_t\}$ and $\{b_t\}$ are recursively characterized by the following difference equations:

$$\begin{cases} a_t = \{(\sigma+1) - \sigma\eta_t\delta a_{t+\Delta}b_t\}^{\frac{-1}{\sigma}}/b_t, & (t < T) \\ b_t = \{1-\eta_t\delta(1-a_{t+\Delta})\}^{-1} & (t < T), \\ a_T = (1+\sigma)^{\frac{-1}{\sigma}};\ b_T = 1. \end{cases} \tag{6}$$

The distribution of $F(v) = v^{\sigma}$ yields that both p_t and c_t are linear in k_t. Intuitively, a_t and b_t capture the monopolist's and the buyer's bargaining power, respectively. Especially, a_0 captures the ex-ante monopoly power. In equilibrium, regardless of the history of prices, i.e., $\{p_0, \cdots, p_{t-\Delta}\}$, the monopolist turns out to choose p_t only based on k_t and the buyer chooses c_t depending only on the current price p_t.

Given the strategies of both players, we are ready to characterize the equilibrium.

Proposition 1. [Equilibrium] *A unique Perfect Bayesian Equilibrium path of $\{(p_t, c_t)\}$ is characterized by* (4), (5), *and* (6).

By straightforward induction, value functions of both players are pinned down as follows.

Corollary 1. [Monopolist's bargaining power and value functions]

The monopolist's and the buyer's value functions V_t and W_t, respectively, are characterized by the monopolist's bargaining power $\{a_t\}$:

$$V_t = \frac{\sigma}{\sigma+1}a_t k_t,\ \ W_t = \{\frac{\sigma}{\sigma+1}(1-\frac{\sigma+2}{\sigma+1}a_t)\}k_t$$

Analogous to prices and cutoffs, the value functions of both the monopolist and the buyer are also linear with respect to the state variable k_t, due to the convenience of a functional form of $F(v) = v^{\sigma}$. The value functions are used to characterize the ex-ante expected surplus of each party: when $t = 0$, V_0 and W_0 captures an ex-ante monopolist's and consumer surplus, respectively, both of which are linear in the ex-ante monopoly power a_0. The formulations of surpluses are utilized in Section 4 and the bargaining efficiency in Section 5.

3 Price Schedules

In this section, I show that the monopolist as well as the buyer make some compromises due to the soft deadline. To rigorously characterize the sizes of such compromises, I extend the framework in the limit case as $\Delta \to 0$, following the techniques of Ausubel and Deneckere (1992) and Fuchs and Skrzypacz (2013) under boundary conditions of the intermediate and terminal deadlines.[24] Note that even when the monopolist's stubbornness to each price decays (i.e., $\Delta \to 0$), a costly delay remains under the hard deadline, evading the Coase conjecture.

Given that the bargaining has a delay, how does the threat of exit change players' behaviors before and after the threat point? I address this question by Lemma 1 and Proposition 2, jointly visualized in Figure 2. One can see that both the cutoff and price schedules discontinuously

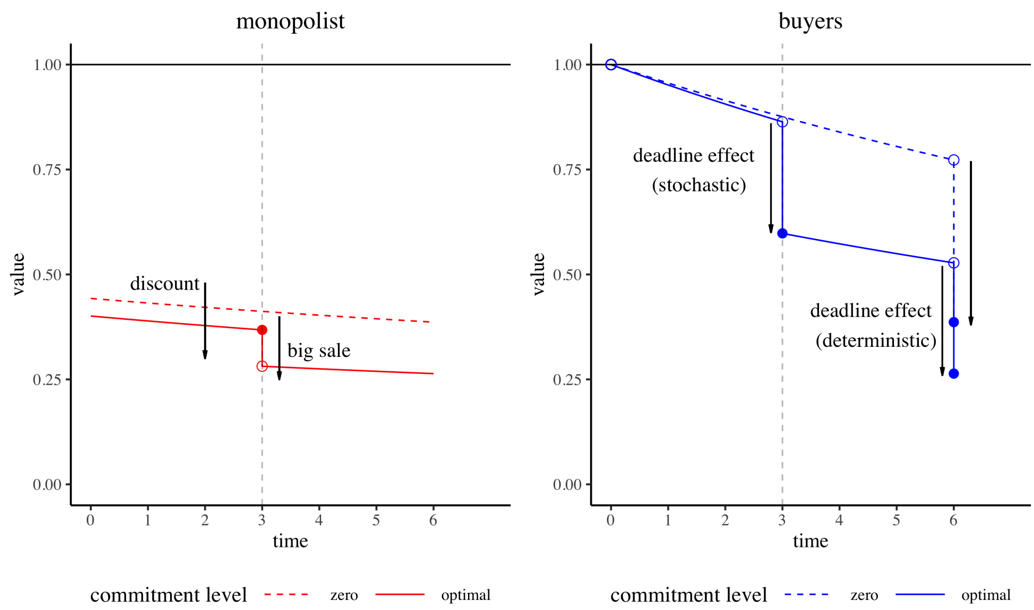

Figure 2: Bargaining dynamics in equilibrium (baseline vs. optimal commitment)
Note: The schedules of the players are simulated in the continuous-time model, where a *soft* deadline with $\alpha = 0.274$ is imposed at $t^* = 3$ out of the time horizon $T = 6$. The bargaining might end at $t^* = 3$ with probability α, but it might continue with probability $1 - \alpha$.

drop around the threat point. As I set $\Delta \to 0$, I analytically pin down the atom of each drop with bargaining primitives γ, σ, α, T and t^*. Before discussing the monopolist's compromise, I start by showing the buyer's compromise at Lemma 1.

Lemma 1. [Deadline effect]

[24] See Figure 7 in Appendix for the construction of variables in detail.

In the limit as $\Delta \to 0$, the buyer's cutoff schedule c_t is continuous at $t \in [0,t_1]$, $t \in [t_d,t_{d+1})$ $(d \in \{1,\cdots,N-1\})$, and $t \in (t_N,T]$, but c_t discontinuously drops at $t = t_d$ $(\forall\, d \in \{1,\cdots,N\})$. Moreover, the cutoff drop at $t = t_d$ is strictly increasing in α_d. $(\forall\, d \in \{1,\cdots,N\})$

[Sketch of Proof] Let us denote that the size of a discontinuous cutoff drop at $t = t_d$ as $g(\alpha_d)$. $g(\alpha_d)$ at $t = t_d$ $(d = 1, \cdots, N)$ is formulated by

$$g(\alpha_d) \equiv \lim_{t \uparrow t_d} c_t - c_{t_d} = (1 - a_{t_d} b_{t_d}) k_{t_d}$$

Note $g(\alpha_d)$ is a function of primitives α_d, σ, and $\lim_{t \downarrow t_d} a_t$, which is determined by σ, γ, t_d, and T. One can check that $g(\alpha_d) \geq 0$ if and only if $\alpha_d \geq 0$, and $g(0) = 0$. (See Appendix for an explicit form of $g(\alpha_d)$ and derivation.) $\square$

Lemma 1 states that the cutoff schedule sharply drops just before the threat point. In the durable good market, an instantaneous *flow* of trade occurs at $t \in [0,t^*)$ and $t \in (t^*,T)$, but an *atom* of trade takes place at $t = t^*$. This is very intuitive: when the buyer faces the "time bomb", she responds by dropping the cutoff sharply. I also show that the commitment (larger α) monotonically expands the drop. The earlier drop can be viewed as a stochastic analog of the *deadline effect* (tested by Roth, Murnighan and Schoumaker (1988) and formalized by Spier (1992) and Fuchs and Skrzypacz (2013)[25]) Thus, the next result follows directly from Lemma 1.

Proposition 2. [Sale after the threat point]

In the limit as $\Delta \to 0$, the monopolist's price schedule p_t is continuous at $t \in [0,t_1]$ and $t \in (t_d,t_{d+1}]$ for all $d \in \{1,\cdots,N-1\}$, $t \in (t_N,T]$, but p_t discontinuously drops at $t = t_d$ $(\forall\, d \in \{1,\cdots,N\})$. Moreover, the price drop at $t = t_d$ is strictly increasing in α_d $(\forall\, d \in \{1,\cdots,N\})$.

[Sketch of the Proof] Let us denote that the size of a discontinuous price drop at $t = t_d$ as

[25] Fuchs and Skrzypacz (2013) consider the limit case of finite-horizon bargaining with asymmetric information and derive an atom trade at the hard deadline.

$h(\alpha_d)$. $h(\alpha_d)$ is formulated by

$$h(\alpha_d) \equiv p_{t_d} - \lim_{t \downarrow t_d} p_t = a_{t_d} k_{t_d} - \lim_{t \downarrow t_d} a_t \lim_{t \downarrow t_d} k_t = \underbrace{\frac{a_{t_d} k_{t_d}}{x_{t^*}} (1 - \lim_{t \downarrow t_d} a_t)}_{>0} \alpha_d$$
$$= \left(\frac{\alpha_d + (1-\alpha_d) \lim_{t \downarrow t_d} a_t}{\alpha_d(\sigma+1) + \lim_{t \downarrow t_d} a_t (1-\alpha_d)}\right)^{\frac{1}{\sigma}} \underbrace{(1 - \lim_{t \downarrow t_d} a_t) k_{t_d}}_{\text{independent of } \alpha_d} \alpha_d.$$

holds. One can see the price drop $h(\alpha_d) \geq 0$ if and only if $\alpha_d \geq 0$, and $h(0) = 0$. This observation implies that when $\alpha_d > 0$, the price schedule discontinuously drops at $t = t_d$. □

Proposition 2 states that when $\alpha \in (0,1)$, an atom of discount occurs if the bargaining survives the threat point. (See Figure 2.) Moreover, the magnitude of the price discount expands with the commitment of the buyer. In fact, this is a direct consequence of the stochastic *deadline effect* in Lemma 1. Because the buyer did not accept before the threat point, the buyer's value is demonstrably bounded. Put differently, her rejection at the threat point is a costly, but effective signal of the lowness of her valuation. Given the limited posterior, the monopolist is forced to sharply discount the price. One may view that the soft deadline serves as a self-screening device of private information at the cost of expected breakdown. Intriguingly, given his post-threat concession, rational backward induction dictates that the monopolist offers a cheaper pre-threat price schedule, formally stated as follows.

Proposition 3. [Discount before the threat point]

There exists some $\overline{\alpha}_d \in (0,1)$ *such that for* $\alpha_d \in (0, \overline{\alpha}_d]$, *the monopolist offers a uniformly lower price* $p_t(\alpha_d) < p_t(\alpha_d = 0)$ *for all* $t \in [0, t_d]$ $\forall\, d \in \{1, \cdots, N\}$.

A proof directly comes from Proof of Proposition 4. (See Appendix) A soft deadline ($\alpha \in (0,1)$) makes a monopolist's cost–benefit accounting qualitatively different from a hard deadline ($\alpha = 1$); the risk-neutral monopolist weighs securing the current expected payoff by myopically leveraging the buyer's compromise at $t \in [0, t^*]$ against an option value of post-threat continuation at $t \in (t^*, T]$. At his optimization problem in the face of the threat, observe that a trade-off emerges between leveraging the buyer's compromise by raising a price (strategic interaction) vs. securing an agreement by discounting a price (price discrimination). As the threat of exit increases, the monopolist projects that the market after the threat point is less lucrative and the post-threat market itself is likely to disappear. Thus, through the self-competition across before- and after-threat point, he may be tempted to discount an opening price to raise the probability of agreement before the threat point.

4 Monopoly Power and Consumer Surplus

Buyers' premium from the soft deadline

Based on the pair of compromises of the monopolist before and after the threat, I present the main result of the study; an optimally designed soft deadline yields a premium to the buyers' group, achieving their maximum expected surplus.

Proposition 4. [Buyers' premium from the soft deadline]

In the limit as $\Delta \to 0$, there exists $\widehat{\alpha_d} \in (0,1)$ that uniquely maximizes W_0 s.t.

$$\widehat{\alpha_d} = \frac{(\lim_{t \downarrow t_d} a_t)^2}{(1 - \lim_{t \downarrow t_d} a_t)(1 + \sigma - \lim_{t \downarrow t_d} a_t)} \tag{7}$$

where $\lim_{t \downarrow t_d} a_t$ is recursively characterized by bargaining primitives γ, T, t_d and α_d for $d = 1, 2, \cdots, N$ s.t.

$$a_T = (1+\sigma)^{\frac{-1}{\sigma}}, \ \lim_{t \downarrow t_d} a_t = e^{-\gamma(T - t_d)} a_{t_{d+1}}, \text{ and } a_{t_{d+1}} = \left(\frac{\{\alpha_{d+1} + (1 - \alpha_{d+1}) \lim_{t \downarrow t_{d+1}} a_t\}^{1+\sigma}}{(1+\sigma)\alpha_{d+1} + (1 - \alpha_{d+1}) \lim_{t \downarrow t_{d+1}} a_t} \right)^{\frac{1}{\sigma}}$$

holds.[26]

[Sketch of the Proof] From Corollary 1, $W_0 = \frac{\sigma}{\sigma+1}(1 - \frac{\sigma+2}{\sigma+1} a_0)$ holds. Differentiating W_0 with respect to α_d, one gets

$$\frac{dW_0}{d\alpha_d} = -\frac{\sigma(\sigma+2)}{(\sigma+1)^2} \frac{da_0}{d\alpha_d} = -\frac{\sigma(\sigma+2)}{(\sigma+1)^2} \frac{da_0}{da_{t_d}} \frac{da_{t_d}}{d\alpha_d}. \tag{8}$$

[26] When $N = 1$, recursive notations are simplified to $\lim_{t \downarrow t^*} a_t = e^{-\gamma(T-t^*)} a_T = e^{-\gamma(T-t^*)} (1+\sigma)^{\frac{-1}{\sigma}}$.

It is sufficient to show $\dfrac{da_0}{da_{t_d}} > 0$ (shown in Appendix) and analyze the impact of α_d on a_{t_d}. Differentiating a_{t_d} with respect to α_d, the first-order condition yields

$$\underbrace{\frac{da_{t_d}(b_{t_d},\alpha_d)}{d\alpha_d}}_{\text{change in monopoly power}} = \underbrace{\underbrace{\frac{da_{t_d}}{db_{t_d}}}_{\text{monopolist's exploitation }(<0)} \underbrace{\frac{db_{t_d}}{d\alpha_d}}_{\text{buyer's compromise }(<0)}}_{\text{strategic interaction}} + \underbrace{\underbrace{\frac{\partial a_{t_d}}{\partial \alpha_d}}_{\text{monopolist's compromise }(<0)}}_{\text{price discrimination}} = 0 \tag{9}$$

Arranging w.r.t. α_d, one gets

$$(\lim_{t\downarrow t_d} a_t)^2(\alpha_d - 1) + \alpha_d\{1+\sigma - \lim_{t\downarrow t_d} a_t(2+\sigma)\} = 0.$$

Solving for α_d, one gets $\widehat{\alpha_d}$. (The detailed derivation and the second-order condition is shown in Appendix.) □

This proposition states that a positive, but imperfect commitment to the soft deadline maximizes the consumer surplus. Recall that the ex-ante consumer surplus W_0 is decreasing in the ex-ante monopoly power a_0 (Corollary 1). The ex-ante monopoly power a_0 is shaped by the monopoly power at the soft deadline a_{t_d} (See the dynamics of the monopoly power for an equation (6)). As such, the non-linearity of the consumer surplus stems from two forces working at the soft deadline: a canonical first-mover advantage of exploitation (strategic interaction; $\frac{da_{t_d}}{db_{t_d}}\frac{db_{t_d}}{d\alpha_d}$) and an irresistible compromise to secure the pre-threat agreement (price discrimination; $\frac{\partial a_{t_d}}{\partial \alpha_d}$). It can be shown that as the soft deadline gets harder, the response from strategic interaction exceeds the one from price discrimination. At the optimal commitment $\widehat{\alpha}$, the two responses are balanced.

If the soft deadline becomes hard ($\alpha = 1$), the optimization is reduced to an ultimatum game without the factor $\frac{\partial a_{t_d}}{\partial \alpha_d}$, and the monopolist exclusively exploits the buyers. Under some moderate threat of exit, however, as the buyers rationally foresee that the monopolist is tempted to price discriminate after the threat point (Proposition 2), he is potentially convinced to concede at the beginning under the self-competition. One may observe that the motive for price discrimination generates a self-competition dynamics previously restrained under the hard deadline.

As the buyer's expected surplus is higher than in the non-commitment case ($\alpha = 0$ or 1), the buyers' group enjoys a premium at the cost of expected breakdown. Figure 3 shows the inverted-U sensitivity of the expected surplus of the demand side.

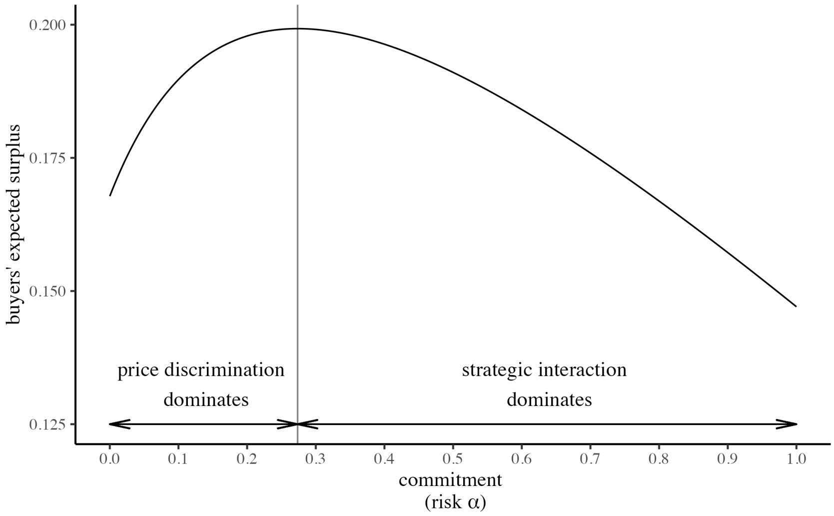

Figure 3: Commitment to the deadline and the buyer's expected surplus

Note: A model is simulated with $T = 6$, $\sigma = 1$, $\gamma = -\ln(0.98)$, and a single threat point is set at $t^* = 3$. The vertical line is the optimal commitment $\widehat{\alpha} = 0.274$.

A simulation under primitives $(T = 6, \sigma = 1, \gamma = -\log(0.98))$ finds that the group's expected surplus is maximized at an interior risk $\widehat{\alpha} = 0.274$. With every rise of 10 $p.p.$ of the risk, when $\alpha < \widehat{\alpha}$, the expected surplus improves by 6.8 $p.p.$ but otherwise deteriorates by 3.6 $p.p.$ (I provide comparative statics of $\widehat{\alpha}$ with primitives σ and γ below.)

What does this mean for the durable goods monopolist literature? As is discussed in the literature review, Coase conjecture negatively relates the durability and monopoly power. The central logic of the conjecture is self-competition dynamics, where the buyer rationally foresees the monopolist's price discrimination and the monopolist loses his commitment on pricing. The conventional wisdom is that a longer bargaining horizon (or interpretably, less durability) hurt the monopoly power. In fact, the buyer's expected surplus under the no-commitment baseline with a single threat point $(N = 1)$ is characterized by

$$W_0 = \mathbb{E}(v)\left(1-(2+\sigma)(1+\sigma)^{-\frac{1+2\sigma}{\sigma}}\exp(-\gamma T)\right),$$

which is strictly increasing in the horizon T.[27] Under the one-shot game ($T \to 0$), the monopolist gains the strongest bargaining power, and the buyer's surplus is minimized at $W_0 = \mathbb{E}(v)(1-(2+\sigma)(1+\sigma)^{-\frac{1+2\sigma}{\sigma}})$. Essentially, the game is reduced to an ultimatum game with

[27] See the Appendix for a derivation.

asymmetric information. In the infinite horizon ($T \to \infty$), by contrast, W_0 increases to $\mathbb{E}(v)$, seizing the maximum efficiency as in the Coase conjecture; the marginal-cost trade occurs with no delay. By stochastically mixing a short and long deadline, the model casts new light on the canonical link between the bargaining horizon and the surplus division; as the commitment to exit increases, the bargaining appears *shorter* in expectation but is favorable for the buyers.

Cost-benefit analysis across the buyer's types

Thus far, I have characterized the optimal commitment on the soft deadline strategy, but the credibility of the soft deadline is a commitment by the buyer's group. Which type of the buyer is better off with an optimal commitment? Are there any buyers getting worse off by the commitment? To see this, I compute the premium across each type of buyer from the commitment. Figure 4 displays the ex-ante surplus of the each type of buyer (top), and the monopolist's expected revenue when facing each type of buyer (bottom). Intriguingly with the

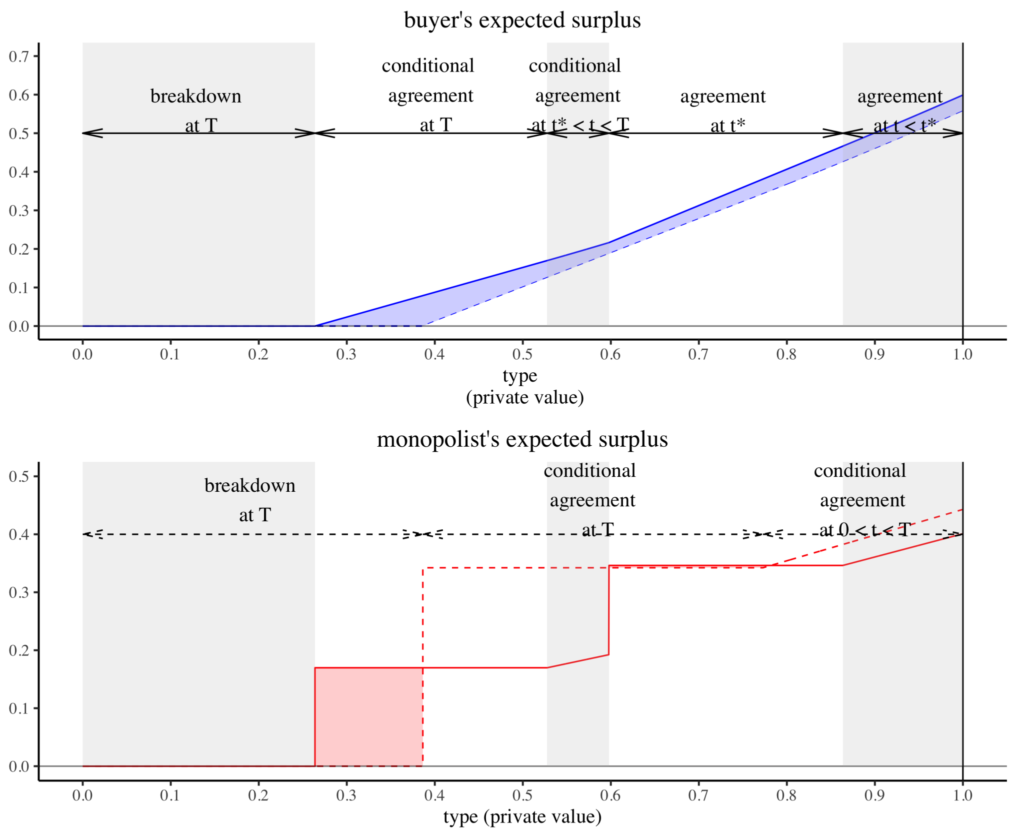

Figure 4: Expected surplus across types (optimal commitment (solid line) vs. baseline (dashed line))

Note: A model is simulated with $T = 6$, $\sigma = 1$, $\gamma = -\ln(0.98)$, and a single threat point is set at $t^* = 3$. A shaded blue or red area is an expected gain from the soft deadline for buyers or the monopolist, respectively.

baseline parameter values ($T=6, \sigma=1, \gamma=-\ln(0.98)$), every type of risk-neutral buyer with $v \geq c_T$ $(=0.26)$ is strictly better off even after taking the termination cost into consideration. One can see that the following cost–benefit analysis of the commitment holds for each segment of buyers. (For notational convenience, denote a hat as the optimal commitment, and 0 as the no-commitment baseline.[28])

- A buyer with $v \in [\widehat{c_{t^*}}, 1]$ is better off with earlier agreement on a cheaper price p_t^* at $t \leq t^*$ with probability 1,
- A buyer with $v \in [\widehat{k_T}, c_{t^*}^*)$ is better off with earlier agreement on a cheaper price p_t^* at $t \in (t^*, T)$ with probability $1-\alpha$.
- A buyer with $v \in [c_T^0, \widehat{k_T})$ is better off with agreement at $t=T$ on a cheaper price p_T^* with probability $1-\alpha$.
- A buyer with $v \in [\widehat{c_T}, c_T^0)$ is better off with agreement at $t=T$ with probability $1-\alpha$, compared to breakdown with probability 1.
- A buyer with $v \in [0, \widehat{c_T})$ is indifferent because they cannot trade for both cases with probability 1.

Overall, this simulation example demonstrates that the soft deadline strategy may be Pareto-improving for every risk-neutral buyer in the demand pool. This assures the participation constraint of each buyer to form a group. In a stark contrast, one can see that a gain for the monopolist (red area) is smaller than the loss (area surrounded by two lines), suggesting the soft deadline serves as a countermeasure to monopoly power.

Comparative statics of optimal commitment

In Proposition 4, I pin down the optimal commitment to the soft deadline. Next, I assess how does the commitment policy varies with other primitives, namely, bargaining friction (discount rate) and the market distribution of private values. To see this, given a parameterized model ($T=6, \sigma=1, \gamma=-\ln(0.98)$), I simulate the sensitivity of $\widehat{\alpha}$ with respect to σ and $\exp(-\gamma)$, as depicted in Figure 5.

[28] Under the simulation with $\gamma=-\ln(0.98)$, $T=6, t^*=3$ at Figure 4, $c_{t^*}^*=0.60, k_T^*=0.53, c_T^0=0.38, c_T^*=0.26$ holds.

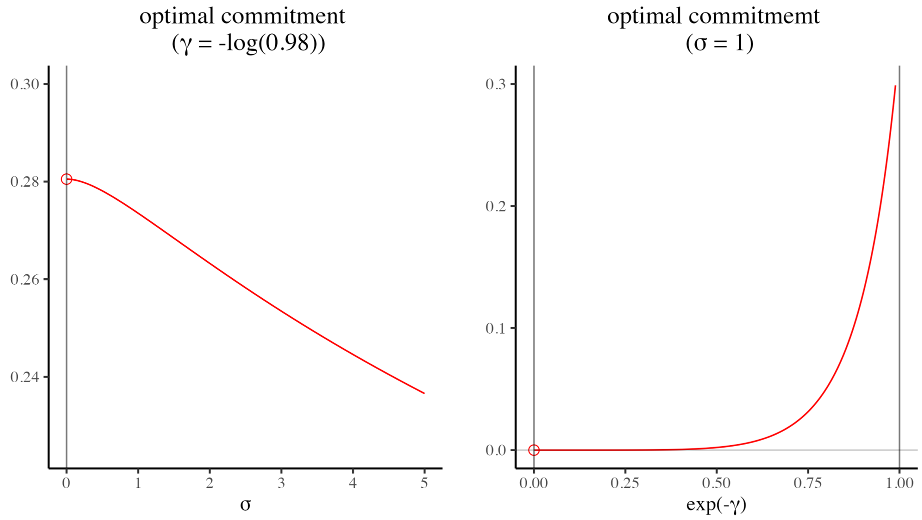

Figure 5: Optimal commitment under various primitives

Note: A baseline model is simulated with $T = 6$, $\sigma = 1$, $\gamma = -\ln(0.98)$, and a single threat point is set at $t^* = 3$. Then, following (7), consider the sensitivity of the optimal commitment $\widehat{\alpha}$ with $\sigma > 0$ or $\exp(-\gamma) > 0$, respectively, all else equal.

One can see that σ and $\exp(-\gamma)$ is positively linked with a lower $\widehat{\alpha}$ (left) and a higher $\widehat{\alpha}$ (right), respectively. To understand the intuition, recall that two forces shaping the response of monopoly power a_0 from an incremental shift of commitment α (or $da_0/d\alpha$) are balanced at $\widehat{\alpha}$. As as α increases, the exploitive response from strategic interaction surpasses compromise response from price discrimination (see the decomposition of change in monopoly power in (9) and Figure 3.) Therefore, the higher $\widehat{\alpha}$ suggests that price discrimination serves stronger relative to strategic interaction at the soft deadline.

I begin with a straightforward case. As a value distribution parameter $\sigma > 0$ increases,[29] the demand pool exhibits more upward-based willingness to pay. As the buyers on average have less incentive to wait, and the monopolist enjoys a larger benefit from strategic interaction, as suggested by the lower $\widehat{\alpha}$ (i.e.; $\frac{d(da_0/d\alpha)}{d\sigma} > 0$). In contrast, as a discount rate $\gamma > 0$ decreases (or equivalently, a periodic discount factor $\exp(-\gamma)$ increases), the bargaining is less frictional and the monopolist suffers from self-competition by a weaker commitment on pricing. The monopolist undergoes a larger loss from price discrimination, as indicated by the higher $\widehat{\alpha}$ (i.e.; $\frac{d(da_0/d\alpha)}{d\gamma} < 0$). The pair of sensitivity tests on optimal commitment is insightful regarding the policy implications of soft deadlines.

[29]Recall that σ captures the first-order stochastic dominance of the cumulative distribution function of private value, $F(v) = v^\sigma$.

5 Efficiency

In the previous section, I discussed the sensitivity of consumer surplus with the credibility of a soft deadline. In the durable goods monopolist model, the consumer surplus is often interchangeably associated with the overall market efficiency. I close my analysis by briefly exploring how the bargaining efficiency responds to commitment intensity. I define the bargaining efficiency $U \equiv V_0 + W_0$ as the sum of the players' ex-ante expected payoffs at $t = 0$ before the private value v is realized. An immediate corollary of Proposition 4 is given as follows.

Corollary 2. [Efficiency impact of the soft deadline] *In the limit as* $\Delta \to 0$, $\widehat{\alpha_d} \in (0,1)$ *uniquely maximizes* U.

[Proof.] From Corollary 1, $V_0 = \frac{\sigma}{\sigma+1} a_0$ and $W_0 = \frac{\sigma}{\sigma+1}(1 - \frac{\sigma+2}{\sigma+1} a_0)$ holds. Therefore,

$$U \equiv V_0 + W_0 = \frac{\sigma}{\sigma+1}(1 - \frac{a_0}{\sigma+1}) \tag{10}$$

holds. Because U is strictly decreasing with a_0, the rest follows the proof of **Proposition 4**. ||

As (10) shows, the efficiency U decreases with an ex-ante monopoly power a_0. As the consumer surplus W_0 is negatively associated with the monopoly power a_0 (Corollary (1)), the model links the consumer surplus to the overall efficiency in the monopolistic market. Therefore, an inverted-U sensitivity of the buyer's expected surplus at Proposition 4 is inherited to the sensitivity of efficiency as well.

Mirrored by the discussion on consumer surplus, this result is also intriguing from the perspective of market design of durable good transactions. Under the classical durable goods monopolist model, a longer trade horizon (or durability) implies larger efficiency. Consider the two extreme cases again as in section 4. Under the one-shot game ($T \to 0$), the bargaining undergoes the largest distortion from the strongest monopoly power with an ultimatum. The bargaining closes instantaneously, but a significant share of buyers cannot buy the goods in the face of the monopoly pricing. In the infinite horizon ($T \to \infty$), however, the bargaining achieves Pareto efficiency, consistent with the Coase conjecture. All the buyers enjoy a competitive pricing with no delay. One may observe that the simple perturbation of deadline structure enhances the efficiency through the resurgence of the self-competition dynamics under the deadline.

6 Concluding Remarks

The monopolist credibly exploits a hard deadline as a commitment device to create his brinkmanship. This paper explores a new commitment technology by a consortium of buyers—a *soft* deadline to counter the monopoly. Using a simple durable goods monopolist model under a deadline, I demonstrate the possibility that the buyers' imperfect commitment to earlier exit potentially augments the buyers' expected surplus by creating a new motive for the monopolist to engage in price discrimination. One may observe that the soft deadline partially revives the dynamics of price discrimination which had been dormant under the hard deadline; when the compromise from price discrimination dominates the exploitation from the strategic interaction at the soft deadline, the imperfect commitment to a soft deadline serves as a countermeasure to monopoly power. This finding revisits conventional wisdom regarding the durable goods monopolist model to relate durability to monopoly power, which lies at the heart of the Coasian logic.

Three caveats are worth noting. First, to adopt a soft deadline strategy, one might consider the best timing and numbers of soft deadlines, upon which this paper is silent.[30] Second, monopolists are plausibly armed with richer outside options than the buyer's side. Nevertheless, the model presumes the common outside options for both parties. Third, although my model imposes risk neutrality for both parties, either party might be risk-averse or risk-loving in the real world, which is potentially associated with their outside options. The distributional implication may depend on the relative strength of outside options and risk preferences. These points are left for future work.

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[30] In a different bargaining protocol for alternating-offer games under complete information, Mauleon and Vannetelbosch (2004) consider a strategic deadline choice to derive a possibility of inefficiency. They remain silent on the surplus division.

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Appendix

Multiple soft deadlines

Instead of the simplest figure with a single soft deadline ($N = 1$), I consider a multiple cases ($N > 0$).

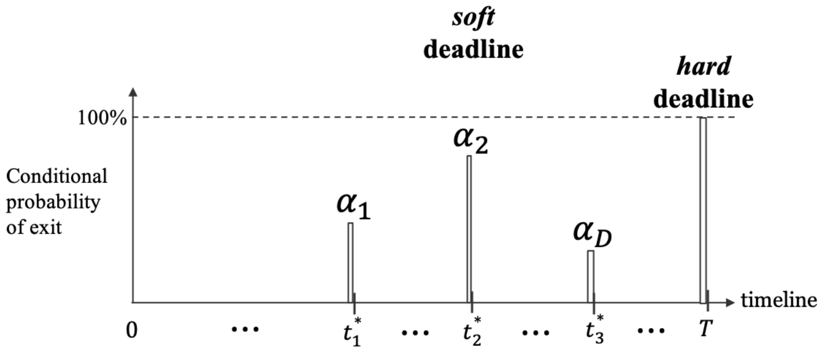

Figure 6: multiple *soft* deadlines

Note: A *soft* deadline with a conditional probability of breakdown $\alpha_d \in (0,1)$ is imposed at the end of time t_d in contrast to the *hard* deadline at time T. The buyer's group commits that the negotiation might end at t_d with probability α_d but continue with probability $1-\alpha_d$.

Discrete period vs. Continuous time

To rigorously illustrate the construction of variables continuous time limit, take the following steps. First, pick up any $t \in [0,T]$ and let $t = m\Delta$, where m is a non-negative integer with $m \in \{1, 2, \cdots, M\}$ and a length of a period $\Delta > 0$. Second, one can define $M \equiv \lceil \frac{T}{\Delta} \rceil$, where $\lceil x \rceil$ is the largest integer that satisfies $\lceil x \rceil \leq x$. Third, as a period of each bargaining round shrinks (i.e., $\Delta \to 0$), variables are defined in the continuous time limit.

Derive bargaining powers $\{a_t\}$ and $\{b_t\}$

To solve the model of (2), (3) and (4), I use backward induction in Sobel and Takahashi (1983).[31] As in (5), price and cutoff schedules are respectively characterized by bargaining powers $\{a_t\}$ and $\{b_t\}$. For analytical convenience, use $x_t \equiv (b_t)^{-1}$ ($\forall t \in [0,T]$) instead of b_t

[31] See the mathematical appendix in Fuchs and Skrzypacz (2013). A detailed proof is available on request.

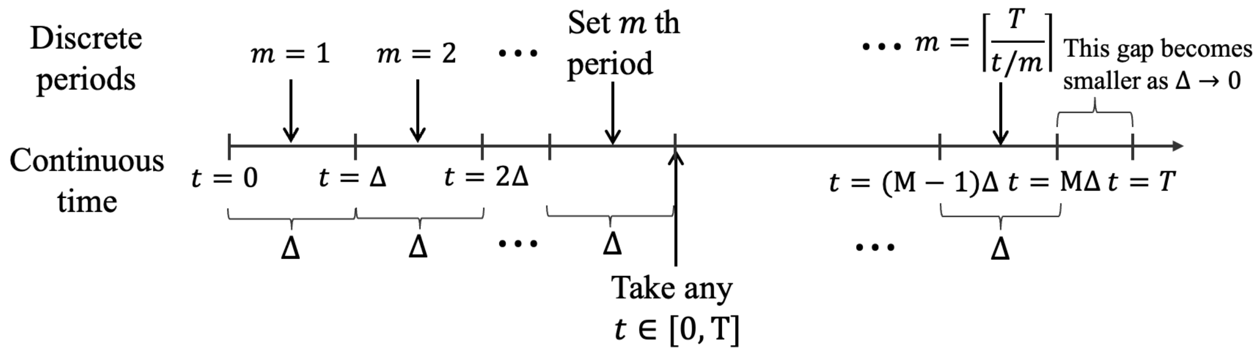

Figure 7: Construction of continuous-time variables

below. Then, I derive a_t by a difference equation of $\{a_t\}$:

$$a_t = \left(\frac{x_t}{(\sigma+1)x_t - \sigma\eta_t \exp(-\gamma\Delta)a_{t+\Delta}}\right)^{\frac{1}{\sigma}} x_t,\ a_T = (1+\sigma)^{\frac{-1}{\sigma}}$$

where $x_t = 1-\eta_t \exp(-\gamma\Delta)(1-a_{t+\Delta})$ and $x_T = 1$. (Recall (6)) Then, translating the difference equation into a differential equation and solving it based on Ausubel and Deneckere (1992) yields

$$\begin{cases} a_t = e^{-\gamma(t_1^*-t)}a_{t_1} & (0 \le t \le t_1) \\ a_t = e^{-\gamma(t_{d+1}-t)}a_{t_{d+1}} & (t_d < t \le t_{d+1}, \quad (d \in \{1,\cdots,N-1\}) \\ a_t = e^{-\gamma(T-t)}a_T & (t_N^* < t \le T), \end{cases} \tag{11}$$

where

$$\begin{aligned} a_{t_d} &= \left(\frac{x_{t_d}}{(1+\sigma)x_{t_d} - \sigma(1-\alpha_d)\lim_{t\downarrow t_d} a_t}\right)^{\frac{1}{\sigma}} x_{t_d} \quad (d \in \{1,\cdots,N\}) \\ &= \left(\frac{(x_{t_d})^{1+\sigma}}{\sigma\alpha_d + x_{t_d}}\right)^{\frac{1}{\sigma}} = \left(\frac{\{\alpha_d + (1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{1+\sigma}}{(1+\sigma)\alpha_d + (1-\alpha_d)\lim_{t\downarrow t_d} a_t}\right)^{\frac{1}{\sigma}} \end{aligned}$$

holds. In the limit as $\Delta \to 0$, I obtain $\{b_t\}$ as

$$\begin{cases} b_{t_d} = \{1-(1-\alpha_d)(1-\lim_{t\downarrow t_d} a_t)\}^{-1} = \{\alpha_d + (1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{-1} & (t_d \ \ (d=1,\cdots,N)) \\ b_t = \{1-(1-a_t)\}^{-1} = \dfrac{1}{a_t} & (t \in [0,t_1) \cup (t_d, t_{d+1}) \cup \ (t_N^*, T)) \\ b_T = 1 & (t=T). \end{cases}$$

Proof of Lemma 1 [Deadline effect]

To derive the cutoff drop at t_d, I characterize k_t by solving a differential equation based on (1). By straightforward algebra,

$$k_t(\Delta) = c_{t-\Delta}(\Delta) = \frac{a_{t-\Delta}(\Delta)k(\Delta)}{x_{t-\Delta}(\Delta)} = \frac{x_{t-\Delta}(\Delta)}{2x_{t-\Delta}(\Delta) - e^{-\gamma\Delta}a_t(\Delta)} k_{t-\Delta}(\Delta)$$

holds. Rearranging it, one gets

$$\frac{k_t(\Delta) - k_{t-\Delta}(\Delta)}{\Delta} = \frac{-x_{t-\Delta}(\Delta) + e^{-\gamma\Delta}a_t(\Delta)}{2x_{t-\Delta}(\Delta) - e^{-\gamma\Delta}a_t(\Delta)} \frac{k_{t-\Delta}(\Delta)}{\Delta}.$$

Thus, I obtain a differential equation such that

$$\frac{dk_t}{dt} \equiv \lim_{\Delta\to 0} \frac{k_t(\Delta) - k_{t-\Delta}(\Delta)}{\Delta} = \lim_{\Delta\to 0} \frac{-\gamma\Delta}{a_t(\Delta)} \frac{k_{t-\Delta}(\Delta)}{\Delta} = -\frac{\gamma}{a_t} k_t.$$

Dividing into time intervals, k_t must satisfy these differential equations.

$$\begin{cases} \dfrac{dk_t}{dt} = -\dfrac{\gamma}{e^{-\gamma(t_1-t)}a_{t_1}} k_t & (0 \le t \le t_1) \\ \dfrac{dk_t}{dt} = -\dfrac{\gamma}{e^{-\gamma(t_{d+1}-t)}a_{t_{d+1}}} k_t & (t_d < t \le t_{d+1}, d \in \{1,\cdots,N-1\}) \\ \dfrac{dk_t}{dt} = -\dfrac{\gamma}{e^{-\gamma(T-t)}a_T} k_t & (t_N^* < t \le T) \end{cases}$$

Solving the three equations above given the boundary conditions k_0, $\lim_{t\downarrow t_d} k_t$ $(d \in \{1,\cdots,N-1\})$,

$\lim_{t \downarrow t_N} k_t$, respectively, one gets

$$\begin{cases} k_t = k_0 \exp\{\frac{e^{\gamma t_1}}{a_{t_1}}(e^{-\gamma t}-1)\} & (0 \leq t \leq t_1) \\ k_t = \lim_{t \downarrow t_d} k_t \exp\{\frac{e^{\gamma(t_{d+1}-t_d)}}{a_{t_{d+1}}}(e^{-\gamma(t-t_d)}-1)\} & (t_d < t \leq t_{d+1}) \\ k_t = \lim_{t \downarrow t_N} k_t \exp\{\frac{e^{\gamma(T-t_N)}}{a_T}(e^{-\gamma(t-t_N)}-1)\} & (t_N < t \leq T), \end{cases} \tag{12}$$

where

$$\lim_{t \downarrow t_d} k_t = c_{t_d} = \frac{a_{t_d}}{x_{t_d}} k_{t_d} = (\frac{x_{t_d}}{\sigma \alpha_d + x_{t_d}})^{\frac{1}{\sigma}} k_{t_d} = \left(\frac{\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t}{(1+\sigma)\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t} \right)^{\frac{1}{\sigma}} k_{t_d} \tag{13}$$

holds. These imply that in the three domains above, k_t is continuous. The cutoff drop $g(\alpha_d)$ at $t = t_d$ ($d \in \{1, \cdots, N\}$) is formulated by

$$\begin{aligned} g(\alpha_d) &\equiv \lim_{t \uparrow t_d} c_t - c_{t_d} = (1 - \frac{a_{t_d}}{x_{t_d}}) k_{t_d} \\ &= \{1 - (\frac{\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t}{(\sigma+1)\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t})^{\frac{1}{\sigma}}\} \exp \frac{\{(\sigma+1)\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t\}^{\frac{1}{\sigma}}(1-e^{\gamma t_d})}{\{\alpha_d + (1-\alpha_d)\lim_{t \downarrow t_d} a_t\}^{\frac{\sigma+1}{\sigma}}}. \end{aligned}$$

One can check that $g(\alpha_d) \geq 0$ if and only if $\alpha_d \geq 0$, and $g(0) = 0$. Note $g(\alpha_d)$ is a function of primitives α_d, σ, and $\lim_{t \downarrow t_d} a_t$, which is determined by σ, γ, t_d, and T. Because $g(0) = 0$, what remains to be shown is for all $\alpha_d \in (0,1)$, $\frac{\partial g(\alpha_d)}{\partial \alpha_d} > 0$. The first-order condition yields that

$$\frac{dg(\alpha_d)}{d\alpha_d} = -\frac{d\lim_{t \downarrow t_d} k_t}{d\alpha_d} = -\frac{d(a_{t_d}/x_{t_d})}{d\alpha_d} k_{t_d} = \frac{\lim_{t \downarrow t_d} a_t \left(\frac{(1-\alpha_d)\lim_{t \downarrow t_d} a_t + \alpha_d}{(1-\alpha_d)\lim_{t \downarrow t_d} a_t + \alpha_d(\sigma+1)} \right)^{\frac{1}{\sigma}} k_{t_d}}{((1-\alpha_d)\lim_{t \downarrow t_d} a_t + \alpha_d)((1-\alpha_d)\lim_{t \downarrow t_d} a_t + \alpha_d(\sigma+1))} > 0$$

holds. The first equality holds because k_{t_d} is independent from α_d. This finding implies the cutoff drop is strictly increasing in α_d. $\|$

Proof of Proposition 2. [Sales after the threat point]

Plugging (11) and (12) into $p_t = a_t k_t$, I obtain the price schedule:

$$
\begin{cases}
p_t = k_0 \exp\{\frac{e^{\gamma(t_1-t)}}{a_{t_1}}(e^{-\gamma t}-1)\}e^{-\gamma(t_1-t)}a_{t_1} & (0 \le t \le t_1) \\
p_t = \lim_{t\downarrow t_d} k_t \exp\{\frac{e^{\gamma(t_{d+1}-t)}}{a_{t_{d+1}}}(e^{-\gamma t}-1)\}e^{-\gamma(t_{d+1}-t)}a_{t_{d+1}} & (t_d < t \le t_{d+1} \ (d \in \{1,\cdots,N-1\})) \\
p_t = \lim_{t\downarrow t_N} k_t \exp\{\frac{e^{\gamma(T-t)}}{a_T}(e^{-\gamma(T-t)}-1)\}e^{-\gamma(T-t)}a_T & (t_N < t \le T).
\end{cases}
$$

Recall that the drop of the price schedule $h(\alpha_d)$ is a function of primitives α_d, σ, and $\lim_{t\downarrow t_d} a_t$, which is determined by σ, γ, t_d, and T. Specifically,

$$
\begin{aligned}
h(\alpha_d) \equiv p_{t_d} - \lim_{t\downarrow t_d} p_t = a_{t_d}k_{t_d} - \lim_{t\downarrow t_d} a_t \lim_{t\downarrow t_d} k_t = \underbrace{\frac{a_{t_d}k_{t_d}}{x_{t_d}}(1-\lim_{t\downarrow t_d} a_t)}_{>0}\alpha_d \\
=\left(\frac{\alpha_d + (1-\alpha_d)\lim_{t\downarrow t_d} a_t}{\alpha_d(\sigma+1) + \lim_{t\downarrow t_d} a_t(1-\alpha_d)}\right)^{\frac{1}{\sigma}} \underbrace{(1-\lim_{t\downarrow t_d} a_t)k_{t_d}}_{\text{independent of } \alpha_d}\alpha_d.
\end{aligned}
$$

Because $h(0) = 0$, what remains to be shown is for all $\alpha_d \in (0,1)$, $\frac{dh(\alpha_d)}{d\alpha_d} > 0$. The first-order condition yields

$$
\frac{dh(\alpha_d)}{d\alpha_d} = (1-\lim_{t\downarrow t_d} a_t)(1+\sigma)\alpha_d^2 + (-1+\sigma)\lim_{t\downarrow t_d} a_t \alpha_d + \lim_{t\downarrow t_d} a_t.
$$

The second-order condition yields

$$
\frac{d^2h(\alpha_d)}{d\alpha_d^2} = 2(1-\lim_{t\downarrow t_d} a_t)(1+\sigma)\alpha_d + (-1+\sigma).
$$

Solving $\frac{d^2h(\alpha_d)}{d\alpha_d^2} = 0$, one gets $\widetilde{\alpha}_d = \frac{1-\sigma}{2(1-\lim_{t\downarrow t_d} a_t)(1+\sigma)}$ such that $\frac{d^2h(\widetilde{\alpha}_d)}{d\alpha_d^2} = 0$. When $\sigma > 1$, $\frac{d^2h(\alpha_d)}{d\alpha_d^2} > 0$ holds. Together with $\frac{dh(0)}{d\alpha_d} = \lim_{t\downarrow t_d} a_t > 0$, for all $\alpha_d \in [0,1]$, $\frac{dh(0)}{d\alpha_d} > 0$ holds. When $0 < \sigma \le 1$, for $\alpha \in [0, \widetilde{\alpha}_d)$, $\frac{d^2h(\alpha_d)}{d\alpha_d^2} < 0$, and for $\alpha \in (\widetilde{\alpha}_d, 1)$, $\frac{d^2h(\alpha_d)}{d\alpha_d^2} > 0$ holds. Then, evaluating $\dfrac{dh(\alpha_d)}{d\alpha_d}$ at the minimizer $\alpha_d = \widetilde{\alpha}_d$, one finds that

$$\begin{aligned}\frac{dh(\widetilde{\alpha}_d)}{d\alpha_d} &= (1-\lim_{t\downarrow t_d} a_t)(1+\sigma)\widetilde{\alpha}_d^2 + (-1+\sigma)\lim_{t\downarrow t_d} a_t \widetilde{\alpha}_d + \lim_{t\downarrow t_d} a_t \\ &= \underbrace{\frac{(1-2\lim_{t\downarrow t_d} a_t)(1-\sigma)^2}{4(1-\lim_{t\downarrow t_d} a_t)(1+\sigma)}}_{>0} + \underbrace{\lim_{t\downarrow t_d} a_t}_{>0} > 0\end{aligned}$$

holds because when $0 < \sigma \leq 1$, $\lim_{t\downarrow t_d} a_t < a_T = (1+\sigma)^{\frac{-1}{\sigma}} < \dfrac{1}{2}$ holds. Therefore, for all $\alpha_d \in (0,1)$, and $\sigma > 0$, $\dfrac{dh(\alpha_d)}{d\alpha_d} > 0$ holds. ||

Proof of Proposition 3. [Discount before the threat point]

First, see the proof of **Proposition 4** below, which shows that $a_{t_d}(\alpha_d = 0) < a_{t_d}(\alpha_d = 1)$ holds (i.e., a shorter bargaining is better-off for a monopolist) and there exists $\widehat{\alpha_d} \in (0,1)$ that minimizes a_{t_d}. Because a_{t_d} is continuous with respect to α_d, by the intermediate value theorem, there must be $\overline{\alpha}_d \in (\widehat{\alpha_d}, 1)$ such that $a_{t_d}(\overline{\alpha}_d) = a_{t_d}(\alpha_d = 0)$. Then, for $\alpha_d \in [0, \overline{\alpha}_d]$, $a_{t_d}(\alpha_d = 0) > a_{t_d}(\alpha_d)$ holds. As $\dfrac{da_0}{da_{t_d}} > 0$, $a_t(\alpha_d = 0) > a_{t_d}(\alpha_d)$ for all $t \in [0, t_d]$. Moreover, as I show $\dfrac{d\lim_{t\downarrow t_d} k_t}{d\alpha_d} < 0$ in **Lemma 1**, $\dfrac{dk_t(\alpha_d)}{d\alpha_d} < 0$ holds for all $\alpha_d \in [0,1)$. (Recall that $\dfrac{dk_t}{d\lim_{t\downarrow t_d} k_t} > 0$ $(t \leq t_d)$ from (12).) Thus, for $\alpha \in (0, \overline{\alpha}]$, $p_t(\alpha = 0) > p_t(\alpha)$ for all $t \in [0, t_d]$. ||

Proof of Proposition 4. [Buyers' premium from the soft deadline]

What remains to be shown is the following two points.

Monotonicity of a_0 w.r.t. a_{t_d}

I show that $\dfrac{da_0}{da_{t_d}} > 0$. For any $d \neq N$, I show a_0 is strictly increasing in a_{t_d}. When $d = 1$,

$$a_0 = e^{-\gamma t_1} a_{t_1} \tag{14}$$

holds. For all $1 < d \leq N$,

$$
\begin{aligned}
a_{t_d} &= \lim_{\Delta \to 0}(1-\delta+\delta A_{t_d+\Delta}(\Delta))\left(\frac{1-\delta+\delta A_{t_d+\Delta}(\Delta)}{(\sigma+1)(1-\delta)+\delta A_{t_d+\Delta}(\Delta)}\right)^{\frac{1}{\sigma}} \\
&= \frac{\{\alpha_d+(1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{\frac{1+\sigma}{\sigma}}}{\{(\sigma+1)\alpha_d+(1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{\frac{1}{\sigma}}}
\end{aligned}
$$

holds. When $\alpha_d > 0$,

$$
\frac{da_{t_d}}{d\lim_{t\downarrow t_d} a_t} = \frac{\{\alpha_d+(1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{\frac{1}{\sigma}}}{\{(\sigma+1)\alpha_d+(1-\alpha_d)\lim_{t\downarrow t_d} a_t\}^{\frac{\sigma+1}{\sigma}}}(1-\alpha_d)\{(1-\alpha_d)\lim_{t\downarrow t_d} a_t+\alpha_d(2+\sigma)\} > 0 \tag{15}
$$

holds. For $t_d < t \leq t_{d+1}$ $(d = 1, \cdots, N-1)$,

$$
\lim_{t\downarrow t_d} a_t = e^{-\gamma(t_{d+1}-t_d)} a_{t_{d+1}} \tag{16}
$$

holds. Combining (14)∼(16) shows a_0 is strictly increasing in a_{t_d}.

Sensitivity of a_{t_d} w.r.t. α_d

Recall that a_{t_d} is the monopolist's bargaining power at the threat point t_d. Using x_t instead of b_t, (9) is reformulated as

$$
\frac{da_{t_d}(x_{t_d},\alpha_d)}{d\alpha_d} = \frac{da_{t_d}}{dx_{t_d}}\frac{dx_{t_d}}{d\alpha_d} + \frac{\partial a_{t_d}}{\partial \alpha_d} = 0. \tag{17}
$$

By some algebra, the monopolist's strategic interaction is captured by

$$
\frac{da_{t_d}}{dx_{t_d}} = (1+\sigma)\alpha_d \frac{x_{t_d}}{\{\lim_{t\downarrow t_d} a_t+\alpha_d(1+\sigma-\lim_{t\downarrow t_d} a_t)\}^{\frac{1+\sigma}{\sigma}}} \quad (>0)
$$

and the buyer's response is captured by $\dfrac{dx_{t_d}}{d\alpha_d} = 1-\lim_{t\downarrow t_d} a_t$ (>0). By contrast, the monopolist's compromise is given by

$$
\frac{\partial a_{t_d}}{\partial \alpha_d} = \lim_{t\downarrow t_d} a_t\{\lim_{t\downarrow t_d} a_t(\alpha_d-1)-\alpha_d\}\frac{x_{t_d}}{\{\lim_{t\downarrow t_d} a_t+\alpha_d(1+\sigma-\lim_{t\downarrow t_d} a_t)\}^{\frac{1+\sigma}{\sigma}}} \quad (>0).
$$

Combining these, (17) is reduced to

$$(\lim_{t\downarrow t_d} a_t)^2(\alpha_d-1)+\alpha_d\{1+\sigma-\lim_{t\downarrow t_d} a_t(2+\sigma)\}=0.$$

Solving for α_d, one gets

$$\widehat{\alpha_d}=\arg\min a_{t_d}=\frac{(\lim_{t\downarrow t_d} a_t)^2}{(1-\lim_{t\downarrow t_d} a_t)(1+\sigma-\lim_{t\downarrow t_d} a_t)} \text{ s.t. } \frac{\partial a_{t_d}(x_{t_d},\widehat{\alpha_d})}{\partial \alpha_d}=0. \tag{18}$$

The second-order condition yields

$$\frac{d^2 a_{t_d}}{d\alpha_d^2}=(\lim_{t\downarrow t_d} a_t)^2(1+\sigma)\{(1-\alpha_d)\lim_{t\downarrow t_d} a_t+\alpha_d\}^{\frac{1-\sigma}{\sigma}} \tag{19}$$

$$\{(1-\alpha_d)\lim_{t\downarrow t_d} a_t+\alpha_d(1+\sigma)\}^{\frac{-1-2\sigma}{\sigma}}>0. \tag{20}$$

I finally show $\widehat{\alpha_d}$ is interior such that $\widehat{\alpha_d}\in(0,1)$. Suppose $\widehat{\alpha_d}\geq 1$; then, by (19),

$$\lim_{t\downarrow t_d} a_t \geq \frac{1+\sigma}{2+\sigma}$$

must hold. Because

$$\lim_{t\downarrow t_d} a_t < A_T=(1+\sigma)^{-\frac{1}{\sigma}}<\frac{1+\sigma}{2+\sigma} \quad (\forall\, \sigma>0)$$

holds, this is a contradiction. Therefore, $\widehat{\alpha_d}<1$ holds. Finally, because $\lim_{t\downarrow t_d} a_t\in(0,1)$ holds, $\widehat{\alpha_d}=\frac{(\lim_{t\downarrow t_d} a_t)^2}{(1-\lim_{t\downarrow t_d} a_t)(1+\sigma-\lim_{t\downarrow t_d} a_t)}>0$ holds. From (18) and (19), a_{t_d} is positively quadratic and minimized with $\widehat{\alpha_d}\in(0,1)$. By (8), W_0 is negatively quadratic and maximized with $\widehat{\alpha_d}\in(0,1)$. ‖

Illustration of strategies of both parties under the optimal commitment

Combining Lemma 1 and Proposition 2, 3,4, I illustrate the cutoff $\{c_t\}$ and price $\{p_t\}$ when $\alpha=0$ vs. $\alpha=\widehat{\alpha}$ in Figure 8.

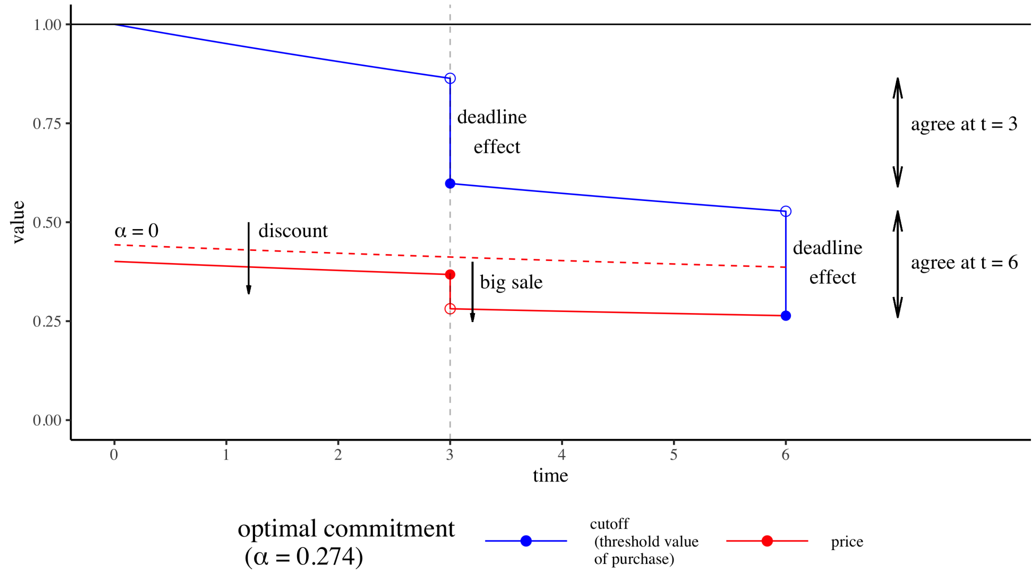

Figure 8: Bargaining dynamics under the optimal commitment

Note: The bargaining might end at å$t^* = 3$ with probability α, but continue with probability $1-\alpha$. The schedules of the players are simulated, where a soft deadline with the optimal commitment intensity $\alpha = 0.274$ is imposed at $t^* = 3$ out of the time horizon $T = 6$. In equilibrium, the cutoff schedule specifies the timing of purchase for a group of buyers of different values. See (5) and (6) for characterization of both schedules.